THE THEORY OF A GYROCOMPASS

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PMM Vol.23, No.5, 1959, pp. 810-817

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(Received 2 June 1959)

In this paper the author investigates the equations of the perturbed motion of a gyrocompass with two rotors, which does not have the properties of the space (free) gyrocompass of Geckeler-Anschutz.

Without dwelling on the properties of a space gyrocompass in detail, we shall mention only that the natural, undamped vibrations of its sensing element have equal periods with respect to all three principal axes of inertia and that this period approximately equals the Schuler period that is $T_0 = 1\pi\sqrt{R/g}$ (R is the earth's radius, g is the gravitational acceleration).

The above mentioned property is imparted through a spring, which couples the two gyroscopes and creates a moment about the vertical axis of each inner ring, given by the formula

$$N = \lambda \sin 2\varepsilon \qquad (0.1)$$

where λ is a certain proportionality factor, 2ϵ is the angle between the axes of rotation of the two rotors. The theory of the space gyrocompass in simplified term is given in the books of Geckeler [1], Grammel [2] and Bulgakov [3].

The equations derived in the work of Ishlinskii [4] could be applied to a gyrocompass which does not possess the properties of a spatial gyrocompass, like, for example, the two-rotor compass of Anschutz, and also certain domestic two-rotor gyrocompasses.

This paper contains an investigation of the unperturbed motion of the above described gyrocompass on the assumption that it is mounted on a ship travelling at high latitudes (70-80°).

In the gyrocompass investigated in this paper the condition (0.1) is not satisfied.

When a gyroscope turns about the axis of its inner ring by a small perturbation angle δ from the position of its unperturbed equilibrium characterized by the angle $\epsilon = \epsilon_0$, then the spring which is coupling the two gyroscopes creates the restoring moment

$$M = s\delta \tag{0.2}$$

where s is the slope of the characteristic of the restoring moment depending on the rigidity of the spring coupling.

1. We shall assume that for the unperturbed motion of a gyrocompass the following condition is satisfied

$$\cos\varepsilon_0 = \frac{Pl}{2Bg}V \qquad (V = \sqrt{(Ru\cos\varphi + v_E)^2 + v_N^2}) \tag{1.1}$$

where B is the angular momentum of a rotor, Pl is the pendular moment of the gyrocompass, V is the velocity of the suspension point of the gyrosphere, u is the angular velocity of the earth, ϕ is the latitude, v_N, v_E are, respectively, the Northern and Eastern components of the ship's velocity.

The differential equations of the perturbed motion of the gyrocompass under consideration, which is mounted on a manoeuvering ship, are given in the paper [4]. Assuming that the conditions (0.2) and (1.1) are satisfied we have

$$\begin{aligned} &\frac{Pl}{g}\frac{d}{dt}(V\alpha) - Pl\,\beta - 2B\sin\varepsilon_0\,\Omega\delta = 0, \qquad && \beta^{\boldsymbol\cdot} + \frac{V\alpha}{R} - \Omega\gamma = 0 \\ &\frac{d}{dt}(2B\sin\varepsilon_0\,\delta) - Pl\,\gamma + \frac{Pl}{g}\,\Omega V\alpha = 0, \qquad && \gamma^{\boldsymbol\cdot} + \frac{s}{2B\sin\varepsilon_0}\,\delta + \Omega\beta = 0 \end{aligned} \tag{1.2}$$

Here α is the aximuth deviation angle of the gyrosphere, β is the elevation angle of the Northern end of the gyrosphere above the plane tangent to the earth's surface, γ is the angle of rotation of the gyrosphere about the North-South line.

The equations (1.2), as well as the equations given in the paper [4], refer to the right-handed coordinate trihedral $x^0y^0z^0$ of Darboux, the x^0-axis coinciding with the velocity vector V of the suspension point along the tangent line to the earth's surface; the earth is assumed to be a sphere of radius R and the z^0-axis is along the normal to the earth's surface.

The angular velocity Ω of the trihedral about the z^0-axis is expressed by the formula

$$\Omega = u\sin\varphi + \frac{v_E}{R}\operatorname{tg}\varphi + \alpha^{*\boldsymbol\cdot} \qquad \left(\alpha^* = \frac{v_N}{Ru\cos\varphi + v_E}\right) \tag{1.3}$$

where α^* is the velocity deviation of the gyrocompass.

2. We shall assume that the ship manoeuvres at a given fixed latitude ϕ.

We shall introduce new variables x_1 and x_4 through the relations

$$\alpha = \frac{Ru\cos\varphi}{V} x_1, \qquad \delta = \frac{\sin\varphi}{\sin\varepsilon_0} x_4. \tag{2.1}$$

We shall also replace the symbols β and γ by the symbols x_2 and x_3, respectively. We shall assume besides that the parameters of the gyrocompass are such that the condition

$$2Bg = PlRu \tag{2.2}$$

is satisfied.

Then the system (1.2) could be reduced to

$$\begin{aligned} &x_1^{\cdot} - \frac{\nu^2}{u\cos\varphi} x_2 - \Omega(t)\,\mathrm{tg}\,\varphi x_4 = 0, \quad x_3^{\cdot} + \frac{p^2(t)}{\nu^2} u \sin\varphi x_4 + \Omega(t) x_2 = 0 \\ &x_2^{\cdot} + u\cos\varphi\, x_1 - \Omega(t) x_3 = 0, \qquad x_4^{\cdot} - \frac{\nu^2}{u\sin\varphi} x_3 + \Omega(t)\,\mathrm{ctg}\,\varphi x_1 = 0 \end{aligned} \tag{2.3}$$

where

$$\nu = \sqrt{\frac{g}{R}}, \qquad p(t) = \frac{\sqrt{Pls}}{2B\sin\varepsilon_0(t)} \tag{2.4}$$

From the system (1.2) we could obtain the equations of motion of the gyrocompass, discussed in [1,2,3].

In order to obtain these equations, we must neglect the terms containing the factor Ω, and we must regard the quantity p as a constant. Under these conditions the system could be broken up into two independent systems of equations with respect to x_1, x_2 and x_3, x_4 which determine the undamped harmonic vibrations of the compass with the circular frequencies ν and p, respectively.

At high latitudes, the terms containing the quantity Ω become of the same order of magnitude as the remaining terms in the system (2.3) and could considerably influence the properties of the solutions. This can be easily shown in the simple case corresponding to Ω and p constant.

With these conditions the characteristic equation of the system (2.3) could be written in the form

$$\lambda^4 + b\lambda^2 + c = 0, \quad (b = p^2 + \nu^2 + 2\Omega^2,\ c = (\Omega^2 - p^2)(\Omega^2 - \nu^2)) \tag{2.5}$$

The necessary and sufficient conditions for the roots of (25) to be pure imaginaries are

$$b > 0, \quad c > 0, \quad b^2 - 4c > 0 \tag{2.6}$$

The first and the third of the above conditions are always satisfied, the second one implies the following inequalities:

$$\text{either } \Omega^2 - p^2 > 0,\ \Omega^2 - \nu^2 > 0, \qquad \text{or} \quad \Omega^2 - p^2 < 0,\ \Omega^2 - \nu^2 < 0 \tag{2.7}$$

For example, if $\nu^2 < \Omega^2 < p^2$, then the equation (2.5) will show that $c < 0$, and the solutions of (2.3) in such a case will grow without bounds.

3. We shall now investigate the case where at a given latitude the ship travels in a circle with constant speed v, beginning to circulate for the first time from the Eastern course. Then

$$v_N = v \sin \omega t, \qquad v_E = v \cos \omega t \qquad \left(\omega = \frac{2\pi}{T}\right) \tag{3.1}$$

where ω is the circular frequency and T is the period.

From the formulas (1.1), (1.3) and (2.2) we have

$$\sin^2 \varepsilon_0 = \left[1 - 2\frac{v_E \cos \varphi}{Ru \sin^2 \varphi} - \left(\frac{v}{Ru \sin \varphi}\right)^2\right] \sin^2 \varphi \tag{3.2}$$

$$\Omega = \frac{v_N^{\cdot}}{Ru \cos \varphi + v_E} + U \sin \varphi + \frac{v_E}{R} \operatorname{tg} \varphi - \frac{v_E^{\cdot}\, v_N}{(Ru \cos \varphi + v_E)^2} \tag{3.3}$$

Using the expressions (3.1) we obtain

$$\sin^2 \varepsilon_0 = \left[1 - 2\frac{v \cos \varphi}{Ru \sin^2 \varphi} \cos \omega t - \left(\frac{v}{Ru \sin \varphi}\right)^2\right] \sin^2 \varphi \tag{3.4}$$

$$\Omega = \frac{v\, \omega \cos \omega t}{Ru \cos \varphi + v \cos \omega t} + u \sin \varphi + \frac{v \operatorname{tg} \varphi}{R} \cos \omega t + \frac{v^2\, \omega \sin^2 \omega t}{(Ru \cos \varphi + v \cos \omega t)^2} \tag{3.5}$$

Under these conditions the variable coefficients of the system (2.3) would be periodic functions of period T. We shall investigate by the Liapunov method the stability of the trivial solution of the system (2.3).

Let $\| x_{jk}(t) \|$ be the fundamental matrix of the solutions of the system (2.3) corresponding to the initial conditions

$$x_{jk}(0) = \delta_{jk} = \begin{cases} 1 & (j = k) \\ 0 & (j \neq k) \end{cases}, \tag{3.6}$$

where j is the index of the function x_j and k is the index of a solution [7].

In our case the characteristic equation of (2.3) will be reciprocal. To prove this we apply the same reasoning which Liapunov used in his investigation of the system of differential equations in the problem of three bodies [5].

We shall present the system (2.3) in the form of two equations in x_1 and x_4. We have

$$\begin{aligned} x_1^{\cdot\cdot} + (\nu^2 - \Omega^2)\, x_1 &= 2\Omega \operatorname{tg} \varphi\, x_4^{\cdot} + \Omega^{\cdot} \operatorname{tg} \varphi x_4 \\ x_4^{\cdot\cdot} + (p^2 - \Omega^2)\, x_4 &= -2\Omega \operatorname{ctg} \varphi x_1^{\cdot} - \Omega^{\cdot} \operatorname{ctg} \varphi x_1 \end{aligned} \tag{3.7}$$

On the strength of formulas (2.4), (3.4), and (3.5) the functions p^2 and Ω will be even and the equations (3.7) will not change if we replace

t by $-t$ and x_4 by $-x_4$. Thus, if ρ is any root of the characteristic equation, then alongside the particular solutions

$$x_1 = M(t)\rho^{t/T}, \qquad x_4 = N(t)\rho^{t/T}$$

where M and N are periodic functions of t of period T, there will also exist particular solutions

$$x_1 = M(-t)\rho^{-t/T}, \qquad x_4 = -N(-t)\rho^{-t/T}$$

The above solutions correspond to the root $1/\rho$ of the same characteristic equation; consequently, the equation will be reciprocal of the form

$$\rho^4 + A_1\rho^3 + A_2\rho^2 + A_1\rho + 1 = 0$$

The regions of stability (non-asymptotic) are determined by inequalities, also derived by Liapunov (see also [6]), which are as follows

$$-2 < A_2 < 6, \qquad 4(A_2 - 2) < A_1^2 < \frac{1}{4}(A_2 + 2)^2 \tag{3.8}$$

The invariants A_1 and A_2 could be computed by the formulas

$$A_1 = -\sum_{j=1}^{4} x_{jj}(T), \qquad A_2 = \sum_{i=1}^{6} L_i(T) \tag{3.9}$$

where

$$L_1 = \begin{vmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{vmatrix}, \qquad L_2 = \begin{vmatrix} x_{11} & x_{13} \\ x_{31} & x_{33} \end{vmatrix}, \qquad L_3 = \begin{vmatrix} x_{11} & x_{14} \\ x_{41} & x_{44} \end{vmatrix}$$
$$L_4 = \begin{vmatrix} x_{22} & x_{23} \\ x_{32} & x_{33} \end{vmatrix}, \qquad L_5 = \begin{vmatrix} x_{22} & x_{24} \\ x_{42} & x_{44} \end{vmatrix}, \qquad L_6 = \begin{vmatrix} x_{33} & x_{34} \\ x_{43} & x_{44} \end{vmatrix} \tag{3.10}$$

4. In order to find the invariants A_1 and A_2, we shall replace the system (2.3) by an equivalent system of Volterra integral equations of the second kind, and then use successive approximations.

The construction of a scheme of successive approximations is more convenient when the ship begins to circulate from the Northern course. Then, instead of (3.1) we have $v_N = v\cos\omega t$, $v_E = -v\sin\omega t$, and the equations (3.2) and (3.3) assume the form

$$\sin^2\varepsilon_0 = [1 + 2\mu\,\mathrm{ctg}^2\varphi\sin\omega t - (\mu\,\mathrm{ctg}\,\varphi)^2]\sin^2\varphi \tag{4.1}$$

$$\Omega = -\frac{\mu\omega\sin\omega t}{1-\mu\sin\omega t} + u\sin\varphi - \mu u\sin\varphi\sin\omega t + \frac{\mu^2\omega\cos^2\omega t}{(1-\mu\sin\omega t)^2}$$

where (4.2)

$$\mu = \frac{v}{Ru\cos\varphi}$$

In this case the functions $p^2(t)$ and $\Omega(t)$ are not even; nevertheless, as we shall see later, the characteristic equation (2.3) is also reciprocal in this case.

We shall make calculations for speeds of the ship less than 25-30 knots (12-15 m/sec), with the period of circulation $T = 4$ min and for $\phi = 80^\circ$.

Under such conditions the dimensionless parameter μ determined from the formula (4.2) will be small compared with unity (of the order 0.15-0.2). Using this assumption in the expansion

$$\frac{1}{1+\mu\sin\omega t} = 1 + \mu\sin\omega t + \ldots$$

and taking into account that $u \sin\phi$ is a small quantity of order μ^2, we obtain

$$\Omega = -\mu\omega\sin\omega t + O(\mu^2) \tag{4.3}$$

where $O(\mu^2)$ denotes all terms of the same order a μ^2 or higher. Similarly, considering $\cot^2\phi$ a small quantity at latitudes 70-80°, we have

$$\sin^2\varepsilon_0 = \sin^2\varphi + O(\mu^2) \tag{4.4}$$

Using only the first terms in the expressions (4.3) and (4.4), we obtain the system (2.3) in the form

$$\begin{aligned} &x_1^{\cdot} - \frac{\nu^2}{u\cos\varphi}x_2 + \mu\omega\,\mathrm{tg}\,\varphi\sin\omega t\,x_4 = 0, \qquad x_2^{\cdot} + u\cos\varphi x_1 + \mu\omega\sin\omega t x_3 = 0\\ &x_4^{\cdot} - \frac{\nu^2}{u\sin\varphi}x_3 - \mu\omega\,\mathrm{ctg}\,\varphi\sin\omega t\,x_1 = 0, \qquad x_3^{\cdot} + \frac{p^2}{\nu^2}u\sin\varphi x_4 - \mu\omega\sin\omega t\,x_2 = 0 \end{aligned} \tag{4.5}$$

Here, on the strength of (4.4), we have taken $p = \sqrt{Pls/2B}\,\sin\phi$.

We shall break up the interval $(0,\ T)$ into two intervals $(0,\ \pi/\omega)$, and $(\pi/\omega\ ,\ 2\pi/\omega)$. We have

$$\sin\omega t = \begin{cases} \dfrac{2}{\pi} - \dfrac{4}{\pi}\left(\dfrac{\cos 2\omega t}{3} + \dfrac{\cos 4\omega t}{15} + \cdots\right) & \left(0 \leqslant t \leqslant \dfrac{\pi}{\omega}\right)\\ -\dfrac{2}{\pi} + \dfrac{4}{\pi}\left(\dfrac{\cos 2\omega t}{3} + \dfrac{\cos 4\omega t}{15} + \cdots\right) & \left(\dfrac{\pi}{\omega} \leqslant t \leqslant \dfrac{2\pi}{\omega}\right) \end{cases} \tag{4.6}$$

In the interval $0 \leqslant t \leqslant \pi/\omega$, using the expansion (4.6), we shall express the system (4.5) in the form

$$\begin{aligned} x_1^{\cdot} - \frac{\nu^2}{u\cos\varphi}x_2 + \Omega_0\,\mathrm{tg}\,\varphi\,x_4 &= f(t)\,\mathrm{tg}\,\varphi x_4\\ x_2^{\cdot} + u\cos\varphi x_1 + \Omega_0 x_3 &= f(t)\,x_3\\ x_3^{\cdot} + \frac{p^2}{\nu^2}u\sin\varphi x_4 - \Omega_0 x_2 &= -f(t)\,x_2\\ x_4^{\cdot} - \frac{\nu^2}{u\sin\varphi}x_3 - \Omega_0\,\mathrm{ctg}\,\varphi x_1 &= -f(t)\,\mathrm{ctg}\,\varphi x_1 \end{aligned} \tag{4.7}$$

where

$$\Omega_0 = \frac{2}{\pi}\mu\omega, \qquad f(t) = \frac{4}{\pi}\mu\omega\left(\frac{\cos 2\omega t}{3} + \frac{\cos 4\omega t}{15} + \cdots\right) \tag{4.8}$$

Considering the right-hand members of the system (4.7) as the known functions of time and applying the method of variation of constants, we shall reduce the system to the equivalent system of Volterra integral equations of the second kind

$$x_j(t) = x_j^0(t) + \int_0^t K_{1,j}(t,\tau)x_1(\tau)\,d\tau + \int_0^t K_{2,j}(t,\tau)x_2(\tau)\,d\tau +$$

$$+ \int_0^t K_{3,j}(t,\tau)x_3(\tau)\,d\tau + \int_0^t K_{4,j}(t,\tau)x_4(\tau)\,d\tau \quad (j=1,\ 2,\ 3,\ 4) \tag{4.9}$$

where the functions $x_j^0(t)$ in the indicated interval are solutions of the system (4.7) when the right-hand members equal zero (homogeneous solutions).

The characteristic equation of this system is of the form (2.5)

$$\lambda^4 + (p^2 + \nu^2 + 2\Omega_0^2)\lambda^2 + (\Omega_0^2 - p^2)(\Omega_0^2 - \nu^2) = 0 \tag{4.10}$$

Using the same reasoning as in Section 2, we conclude that if the quantity Ω_0^2 is in the interval

$$\nu^2 < \Omega_0^2 < p^2 \tag{4.11}$$

then the equation (4.10) would have a positive root.

The condition $\Omega_0 > \eta$ could be reduced to

$$v > \frac{\pi}{2}\frac{T}{T_0}Ru\cos\varphi \tag{4.12}$$

Assuming that the condition (4.11) is satisfied, we obtain the roots of the equation (4.10)

$$\lambda_1 = m, \qquad \lambda_2 = -m, \qquad \lambda_3 = qi, \qquad \lambda_4 = -qi \tag{4.13}$$

We shall apply to the equations (4.9) an iteration scheme, taking as the first approximation $x_j(t) = x_j^0(t)$. The iteration scheme is fully legitimate, because the sequence of the successive approximations for the equations (4.9) is convergent.

Using only the first approximation we obtain solutions in the interval $(0, \pi/\omega)$ in the form

$$x_j = C_j \operatorname{ch} mt + D_j \operatorname{sh} mt + E_j \cos qt + G_j \sin qt \tag{4.14}$$

Here

$$
\begin{aligned}
C_1 &= \frac{1}{m^2+q^2}\left[(m^2+p^2+\Omega_0^2)\,x_1(0) - 2\frac{\nu^2}{u\cos\varphi}\,\Omega_0 x_3(0)\right] \qquad (4.15)\\
D_1 &= \frac{1}{m(m^2+q^2)}\left[\frac{\nu^2}{u\cos\varphi}(m^2+p^2-\Omega_0^2)\,x_2(0) - \Omega_0(m^2+\Omega_0^2-p^2)\operatorname{tg}\varphi x_4(0)\right]\\
E_1 &= -\frac{1}{m^2+q^2}\left[(p^2+\Omega_0^2-q^2)\,x_1(0) - 2\frac{\nu^2}{u\cos\varphi}\,\Omega_0 x_3(0)\right]\\
G_1 &= -\frac{1}{q(m^2+q^2)}\left[\frac{\nu^2}{u\cos\varphi}(p^2-\Omega_0^2-q^2)\,x_2(0) - \Omega_0(\Omega_0^2-p^2-q^2)\operatorname{tg}\varphi\, x_4(0)\right]\\
C_2 &= \frac{1}{m^2+q^2}\left[(m^2+p^2+\Omega_0^2)\,x_2(0) + \Omega_0\left(\frac{p^2}{\nu^2}+1\right)u\sin\varphi x_4(0)\right]\\
D_2 &= \frac{1}{m(m^2+q^2)}\left[u\cos\varphi\left(\Omega_0^2\frac{p^2}{\nu^2} - m^2 - p^2\right)x_1(0) - \Omega_0(m^2+\Omega_0^2-\nu^2)\,x_3(0)\right]\\
E_2 &= -\frac{1}{m^2+q^2}\left[(\Omega_0^2+p^2-q^2)\,x_2(0) + \Omega_0\left(\frac{p^2}{\nu^2}+1\right)u\sin\varphi x_4(0)\right]\\
G_2 &= -\frac{1}{q(m^2+q^2)}\left[u\cos\varphi\left(\Omega_0^2\frac{p^2}{\nu^2} + q^2 - p^2\right)x_1(0) - \Omega_0(\Omega_0^2-\nu^2-q^2)\,x_3(0)\right]\\
C_3 &= \frac{1}{m^2+q^2}\left[-\frac{u\cos\varphi}{\nu^2}\,\Omega_0(p^2+\nu^2)\,x_1(0) + (m^2+\Omega_0^2+\nu^2)\,x_3(0)\right]\\
D_3 &= \frac{1}{m(m^2+q^2)}\left[\Omega_0(m^2+\Omega_0^2-p^2)\,x_2(0) + \left(\Omega_0^2 - p^2 - \frac{p^2}{\nu^2}m^2\right)u\sin\varphi x_4(0)\right]\\
E_3 &= \frac{1}{m^2+q^2}\left[\frac{u\cos\varphi}{\nu^2}\,\Omega_0(p^2+\nu^2)\,x_1(0) - (\Omega_0^2-\nu^2+q^2)\,x_3(0)\right]\\
G_3 &= -\frac{1}{q(m^2+q^2)}\left[\Omega_0(\Omega_0^2-p^2-q^2)\,x_2(0) + \left(\Omega_0^2 - p^2 + \frac{p^2}{\nu^2}q^2\right)u\sin\varphi x_4(0)\right]\\
C_4 &= \frac{1}{m^2+q^2}\left[2\Omega_0\frac{\nu^2}{u\sin\varphi}\,x_2(0) + (m^2+\Omega_0^2+\nu^2)\,x_4(0)\right]\\
D_4 &= \frac{1}{m(m^2+q^2)}\left[-\Omega_0(\nu^2-m^2-\Omega_0^2)\operatorname{ctg}\varphi x_1(0) + \frac{\nu^2}{u\sin\varphi}(m^2-\Omega_0^2+\nu^2)\,x_3(0)\right]\\
E_4 &= -\frac{1}{m^2+q^2}\left[2\Omega_0\frac{\nu^2}{u\sin\varphi}\,x_2(0) + (\Omega_0^2+\nu^2-q^2)\,x_4(0)\right]\\
G_4 &= \frac{1}{q(m^2+q^2)}\left[\Omega_0(\nu^2+q^2-\Omega_0^2)\operatorname{ctg}\varphi x_1(0) - \frac{\nu^2}{u\sin\varphi}(\nu^2-\Omega_0^2-q^2)\,x_3(0)\right]
\end{aligned}
$$

Having formulas (4.15), it is easy to construct the matrix $\|x_{jk}(t)\|$, satisfying the initial conditions (3.6).

Taking into account the conditions (3.6) and using formulas (4.14) and (4.15), we can construct the solutions x_{jk} in the interval $(0, \pi/\omega)$, which after proper adjustment to the new initial conditions can be continued in the interval $(\pi/\omega, 2\pi/\omega)$.

Equation (4.10) does not change when Ω_0 is replaced by $-\Omega_0$; therefore, in the interval $(\pi/\omega, 2\pi/\omega)$ we shall also have solutions of the form

(4.14), in which the coefficients C, D, E, G should be computed by the formulas (4.15) changing the sign of Ω_0.

Let $\phi = 80^0$, $\nu = 30$ knots, $T = 4$ min. The parameters of the compass are as follows:

$$Pl = 4550 \text{ gcm}; \quad s = 200 \text{ gcm}; \quad 2B = 21 \times 10^4 \text{ gcm sec.}$$

For the above data we have

$$p^2 = 2.0330 \times 10^{-5} \text{ sec}^{-2}; \ \Omega_0^2 = 1.02438 \times 10^{-5} \text{ sec}^{-2}.$$

Since $\nu^2 = g/R = 0.15376 \times 10^{-5}$ sec^{-2}, the condition (4.11) is satisfied, and the roots of the equation will have the form (4.13).

In this case the computed values of m and q are

$$m = 1.405799 \times 10^{-3} \text{ sec}^{-1}; \quad q = 6.655935 \times 10^{-3} \text{ sec}^{-1}.$$

For the above data the matrix $\|x_{js}(T)\|$, satisfying the initial conditions (3.6), has the form

$$\left\| \begin{array}{rrrr} 0.932136 & 28.392466 & -1.123568 & -0.491128 \\ -0.00456165 & 0.932136 & -0.0909062 & -0.031682 \\ -0.00558646 & -0.0865987 & 0.492616 & -0.177823 \\ -0.0160291 & -0.198115 & 4.249876 & 0.492616 \end{array} \right\| \tag{4.16}$$

The characteristic equation of the above matrix written with the accuracy of three decimal figures is

$$\rho^4 - 2.849\rho^3 + 3.805\rho^2 - 2.849\rho + 1.000 = 0. \tag{4.17}$$

Equations (4.5) have also been integrated on the high speed computer "STRELA" under the same conditions that lead to the matrix (4.16). The characteristic equation computed by the machine was

$$\rho^4 - 2.848\rho^3 + 3.809\rho^2 - 2.848\rho + 1.000 = 0 \tag{4.18}$$

Equation (4.18) agrees very well with equation (4.17).

It can easily be shown that in both cases the conditions (3.8) are satisfied; hence, the solutions are stable in the Liapunov sense, in spite of the presence of the hyperbolic functions in the expressions (4.14).

5. If we consider the equations of the perturbed motion in the finite interval of time $(0, t^*)$ (for example, in the interval $0 < t < \pi/\omega$, which corresponds to half a circulation) after which the ship resumes the straight line course, then the presence of a positive root in the equation (4.10) causes the coordinates x_j to increase in the given interval (the initial values at $t = t_0$ are $x_j(0)$).

In cases of prolonged manoeuvres of the ship, which consist of sequences of turns and circulations, separated by intervals in which the ship travels in a straight line with constant speed (in such a case the motion of a gyrocompass is described by a system of equations different from (4.5)), and with non-zero initial conditions the gyrocompass could acquire very undesirable oscillations of increasing amplitude.

6. Now considering the equation (4.10) and the formulas (2.4) we shall allow the slope s characterizing the restoring moment to be sufficiently small, and satisfy the condition $p = \nu$.

It could easily be shown that in such a case the indicated equation will not have positive roots, consequently there will be no tendency for oscillations to increase in amplitude.

It could be mentioned, that when $p = \nu$, the equations of the perturbed motion (2.3) can be integrated in closed form and will possess the properties of the space gyrocompass of Geckeler-Anschutz.

This could easily be demonstrated by constructing the solution of (2.3) by the method used in [4].

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Translated by T.L.